Fault detection for LPV systems: Loop shaping $\mathcal{H}_-$ approach

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Abstract: This paper addresses a method for fault detection (FD) in linear parameters varying (LPV) systems by maximizing the fault to residual sensitivity. It uses the newly developed $\mathcal{H}_-$ index properties and minimizing the well known $\mathcal{H}_\infty$ norm for worst case uncertainties and disturbance attenuation. A loop shaping approach for the $\mathcal{H}_-$ FD problem is proposed.
The multi-objectives problem is formulated as Linear Matrix Inequalities (LMI) problem for polytopic systems. An application on lateral vehicle dynamics is given as an illustrative example for this approach.

Keywords: Residual generation, fault detection, LPV systems, $\mathcal{H}_-$, $\mathcal{H}_\infty$, LMI.

1. INTRODUCTION

Fault detection have received significant interest in both research and application domains since the last two decades. Among all concrete systems, automotive and aerospace applications represent a wide field of interest in terms of diagnosis (Varrier et al., 2014). Highly equipped with sensors and actuators, fault detection have become one essential issue for reliability and safety of these systems.

In comparison with analytical and soft computing approaches (such as neural networks and set-membership approaches), model based approaches including observer based design or parity space methods are powerful tools when plant dynamical equations are available (Chen and Patton, 1999; Liu and Patton, 1998; Ding, 2013; Isermann, 2005). Trying to enhance the robustness to unknown inputs and sensitivity for faults, many performance indexes has been introduced and developed: H_∞, H_-, mixed H_∞/H_- (Hou and Patton, 1996; Liu et al., 2005; Wang et al., 2007; Chadli et al., 2011). Such criteria are often in the LMI control framework (Boyd et al., 1994).

On the other hand, LPV modeling offers an interesting framework to model non-linear plants with parameter variations, that are treated as linear systems with not necessarily known but on-line measurable, time-varying parameters. LPV-control has evolved rapidly and have been employed in many control applications (Mohammadpour and Scherer, 2012).

Hence, fault detection for LPV system has recently gained much attention. Authors proposed algebraic approach, model redundancy (Bokor and Balas, 2004; Seron and Don, 2015). In (Chadli et al., 2011) an optimization criterion for the $\mathcal{H}_-/\mathcal{H}_\infty$ objective is proposed.

In this paper, an observer based filter is designed with the mixed $\mathcal{H}_-/\mathcal{H}_\infty$ and pole placement objectives:

- the $\mathcal{H}_-$ index is used to enhance the residual to fault sensitivity,
- the $\mathcal{H}_\infty$ norm is used to attenuate the residual to disturbances effects,
- pole placement region is used to tune the time response of the FD observer.

The desired observer is computed by solving a set of LMIs in the framework of the quadratic stability for LPV polytopic systems. A compromise between fault sensitivity, unknown input rejection and eigen region assignment is optimized via a convex optimization algorithm.

The outline of this paper is as follows. Problem formulation is given in Section II. In section III, preliminaries for the synthesis of $\mathcal{H}_\infty$ observer, $\mathcal{H}_-$ fault detector, and pole placement technique. Fault detection observer scheme is given in Section IV using loop shaping by additive/multiplicative filter design. A minimization and maximization criteria are used to solve an optimization problem set by the LMIs. The above results are illustrated by an example with application to lateral vehicle dynamics in Section V. Finally, Section VI shows the concluding remarks and the possible future work.

Notations: Notations used in this paper are standard. X^T is the transposed of matrix X, the star symbol $(\star)$ in a symmetric matrix denotes the transposed block in the symmetric position. The notation $P > (<) 0$ means P is real positive (negative) definite matrix. 0 and I denote zeros and identity matrix of appropriate dimensions.

2. PROBLEM FORMULATION

Consider the state space representation of a LPV system:

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) \\ \qquad +E_d(\rho(t))d(t) + E_f(\rho(t))f(t) \\ y(t) = Cx(t) + Du(t) + F_d d(t) + F_f f(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the measurement output vector, $u \in \mathbb{R}^m$ is the input vector, $d \in \mathbb{R}^{n_d}$ is the disturbance vector, $f \in \mathbb{R}^{n_f}$ is the vector of faults to be detected.

$\rho(.)$ is a varying parameter vector that takes values in the parameter space $\mathcal{P}_\rho$ such that:

$\mathcal{P}_\rho := \{\rho(t) \triangleq [\rho_1(t)\ \rho_2(t)\ \dots\ \rho_l(t)]^T \in \mathbb{R}^l$ and $\rho_i(.) \in [\underline{\rho}_i\ \overline{\rho}_i]\ \forall i = 1, \dots, l\}$

In the following the subscript t is omitted without confusion for simplification.

Assuming that $\mathcal{P}_\rho$ is a convex hull, the system is rewritten into a polytopic description:

$$\left[\begin{array}{c|ccc} A(\rho) & B(\rho) & E_d(\rho) & E_f(\rho) \\ \hline C & D & F_d & F_f \end{array}\right] = \sum_{i=1}^{N} \alpha^i(\rho) \left[\begin{array}{c|ccc} A(\omega_i) & B(\omega_i) & E_d(\omega_i) & E_f(\omega_i) \\ \hline C & D & F_d & F_f \end{array}\right] \quad (2)$$

with α^i is a scheduling parameter that lies in a convex set:

$$\Psi = \quad \{\alpha^i(\rho) \in \mathbb{R}^N, \alpha^i(\rho) = [\alpha^1(\rho), ..., \alpha^N(\rho)]^T, \alpha^i(\rho) \geq 0, \forall j, \sum_{i=1}^{N} \alpha^i(\rho) = 1\} \quad (3)$$

with $N = 2^l$ is the number of vertices of the polytope. Define $\mathbb{S} = 1, \dots, N$, and denote $X_i \triangleq X(\omega_i)$. The matrices A_i, B_i, $E_{d,i}$, $E_{f,i}$, C, D, F_d and F_f are the nominal system matrices at each i-th vertex, known and of appropriate dimensions.

Consider the following residual generator observer:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{N} \alpha^i(\rho) \left[A_i \hat{x} + B_i u + L_i (y - \hat{y})\right] \\ \hat{y} = \sum_{i=1}^{N} \alpha^i(\rho) \left[C\hat{x} + Du\right] \\ r = \sum_{i=1}^{N} \alpha_i(\rho)[y - \hat{y}] \end{cases} \quad (4)$$

In all what follows, for simplicity, denote $X_\rho \triangleq X(\rho)$.

Define the state error at each vertex as $\tilde{x} = x - \hat{x}$. Then:

$$\begin{aligned} \dot{\tilde{x}} &= (A_\rho - L_\rho C)\tilde{x} + (E_{d_\rho} - L_\rho F_d)d \\ &\quad + (E_{f_\rho} - L_\rho F_f)f \\ r &= C\tilde{x} + F_d d + F_f f \end{aligned} \quad (5)$$

It can be put in the form:

$$\begin{cases} \dot{\tilde{x}} = A^*_\rho \tilde{x} + E^*_{d_\rho} d + E^*_{f_\rho} f \\ r = C\tilde{x} + F_d d + F_f f \end{cases} \quad (6)$$

with $A^*_\rho = A_\rho - L_\rho C$, $E^*_{d_\rho} = E_{d_\rho} - L_\rho F_d$, and $E^*_{f_\rho} = E_{f_\rho} - L_\rho F_f$.

The objective of the $\mathcal{H}_-/\mathcal{H}_\infty$ FD observer is resumed by the following conditions:

$$\sup_{d \in \mathbb{R}^{n_d}} \frac{\|r\|_2}{\|d\|_2} < \gamma \quad (\mathcal{H}_\infty) \quad (7)$$

$$\inf_{f \in \mathbb{R}^{n_f}} \frac{\|r\|_2}{\|f\|_2} > \beta \quad (\mathcal{H}_-) \quad (8)$$

The problem is formulated as following: $\forall i \in \mathbb{S}$, find the matrices L_i that maximize β and minimize γ such that the conditions (7)-(8) are satisfied and the FD observer is stable.

Assumption 1. In this study the pair (A_ρ, C) is assumed observable, or without loss of generality is detectable. It is a standard assumption for all fault detection problems.

Assumption 2. In this study, it is assumed that faults are spectrally located in low frequencies (offsets, low time varying failures). However, both sensor faults and actuator faults are considered.

Remark 1. In the problem formulation, the C matrix is assumed parameter independent. However this approach can be generalized in the case of parameter dependent matrix $C(\rho)$, by adding the strictly proper filter to the output (Apkarian et al., 1995):

$$\mathcal{F}_y : \begin{bmatrix} \dot{x}_f \\ y_f \end{bmatrix} = \left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & 0 \end{array}\right] \begin{bmatrix} x_f \\ y \end{bmatrix} \quad (9)$$

By interconnecting the two systems, it is obtained:

$$\left[\begin{array}{c} \dot{x} \\ \dot{x}_f \\ \hline y_f \end{array}\right] = \left[\begin{array}{cc|ccc} A(\rho) & 0 & B(\rho) & E_d(\rho) & E_f(\rho) \\ B_f C(\rho) & A_f & B_f D & B_f F_d & B_f F_f \\ \hline 0 & C_f & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} x \\ x_f \\ \hline u \\ d \\ f \end{array}\right] \quad (10)$$

Then in the new extended system, the output matrix is ρ independent.

3. PRELIMINARIES

3.1 $\mathcal{H}_\infty$ synthesis problem

Theorem 1. For a given LPV system with faults as defined in (1), $\forall i \in \mathbb{S}$, if there exists matrices U_i, a symmetric matrix $P > 0$ and positive scalar γ, such that minimizing γ subject to the following inequalities :

$$\left[\begin{array}{c|c} \begin{matrix} PA_i + A_i^T P \\ +U_i C + C^T U_i^T + C^T C \end{matrix} & \begin{matrix} PE_{d_i} + U_i F_d \\ +C^T F_d \end{matrix} \\ \hline \star & F_d^T F_d - \gamma^2 I \end{array}\right] < 0 \quad (11)$$

Then an $\mathcal{H}_\infty$ observer can be designed where the gain filter at the i-th vertex is $L_i = -P^{-1} U_i$

Proof 1. Considering the candidate common Lyapunov function $V = \tilde{x}^T P \tilde{x} > 0$, and combining the $\mathcal{H}_\infty$ unknown inputs rejection condition in (7) to the stability sufficient condition in the fault free case ($\dot{V}|_{f=0} < 0$), the following condition could be written:

$$\dot{V} + r^T r - \gamma^2 d^T d < 0 \quad (12)$$

Denote $U_\rho \triangleq -PL_\rho$, then (12) is equivalent to:

$$
\begin{aligned}
&(A_\rho^*\tilde{x} + E_{d_\rho}^* d)^T P\tilde{x} + \tilde{x}^T P(A_\rho^*\tilde{x} + E_{d_\rho}^* d) \\
&+(C\tilde{x} + F_d d)^T(C\tilde{x} + F_d d) - \gamma^2 d^T d \qquad\qquad < 0 \\
\Leftrightarrow &\tilde{x}^T(PA_\rho^* + A_\rho^{*T}P + C^TC)\tilde{x} \\
&+2\tilde{x}^T(PE_{d_\rho}^* + C^TF_d)d + d^T(F_d^TF_d - \gamma^2 I)d < 0 \quad (13) \\
\Leftrightarrow &\tilde{x}^T(PA_\rho + A_\rho^TP + U_\rho C + C^TU_\rho^T + C^TC)\tilde{x} \\
&+2\tilde{x}^T(PE_{d_\rho} + U_\rho^TF_d + C^TF_d)d \\
&+d^T(F_d^TF_d - \gamma^2 I)d \qquad\qquad < 0
\end{aligned}
$$

Thus, putting (13) in the quadratic form :

$$
\begin{bmatrix} \tilde{x} \\ d \end{bmatrix}^T
\left[\begin{array}{c|c}
\begin{array}{c} PA_\rho + A_\rho^TP \\ +U_\rho C + C^TU_\rho^T \\ +C^TC \end{array} & \begin{array}{c} PE_{d_\rho} + U_\rho F_d \\ +C^TF_d \end{array} \\
\hline
\star & F_d^TF_d - \gamma^2 I
\end{array}\right]
\begin{bmatrix} \tilde{x} \\ d \end{bmatrix} < 0 \quad (14)
$$

This inequality hold $\forall \left[\tilde{x}^T \ d^T\right]^T \neq 0$, and using the assumption that the system is under a polytopic state space representation, the above inequality stands if the following LMIs hold $\forall i \in \mathbb{S}$:

$$
\left[\begin{array}{c|c}
\begin{array}{c} PA_i + A_i^TP \\ +U_iC + C^TU_i^T + C^TC \end{array} & \begin{array}{c} PE_{d_i} + UF_d \\ +C^TF_d \end{array} \\
\hline
\star & F_d^TF_d - \gamma^2 I
\end{array}\right] < 0 \quad (15)
$$

□

3.2 $\mathcal{H}_-$ synthesis problem

Theorem 2. For a given LPV system with faults as defined in (1), $\forall i \in \mathbb{S}$, if there exists matrices U_i, a symmetric matrix $P > 0$ and positive scalar β, such that maximizing β subject to the following inequality :

$$
\left[\begin{array}{c|c}
\begin{array}{c} PA_i + A_i^TP \\ +U_iC + C^TU_i^T - C^TC \end{array} & \begin{array}{c} PE_{f_i} + U_iF_f \\ -C^TF_f \end{array} \\
\hline
\star & -F_f^TF_f + \beta^2 I
\end{array}\right] < 0 \quad (16)
$$

Then an $\mathcal{H}_-$ observer can be designed where the gain filter at the i-th vertex is $L_i = -P^{-1}U_i$

Proof 2. Using the same Lyapunov function as in disturbance free case and the $\mathcal{H}_-$ fault sensitivity property from (8), it follows:

$$\dot{V}|_{d=0} - r^Tr + \beta^2 f^Tf < 0 \quad (17)$$

And following the steps of calculations as in the proof of *Theorem 1*, the LMI in (16) is straightforward. □

Remark 2. The LMI proposed in this theorem states only sufficient condition to satisfy the $\mathcal{H}_-$ problem. In (Liu et al., 2005), it has been proven that solution of (8) does not require the matrix P to be sign defined. However in the case of the multi-objectives design, the condition $P > 0$ becomes necessary condition for the $\mathcal{H}_\infty$ problem.

3.3 Eigen region assignment

Theorem 3. For a given square $n \times n$ matrix A_ρ that respects the assumption of the polytopic framework, if there exists a symmetric matrix $P > 0$ and a scalar Ω_{max} such that $\forall i \in \mathbb{S}$ the following inequalities hold:

$$A_i^TP + PA_i - 2\Omega_{max}P < 0 \quad (18)$$

Then all eigenvalues of A_ρ are on left half plane of Ω_{max}.(Chilali and Gahinet, 1996)

Proof 3. (18) is a result of a classical Lyapunov function for sufficient condition of stability.

$\dot{x} = (A_\rho - \Omega_{max}I)x$ is stable if there exist a symmetric matrix $P > 0$ where $V = x^TPx$, $\dot{V} < 0$. Thus

$$(A_\rho - \Omega_{max}I)^TP + P(A_\rho - \Omega_{max}I) < 0 \quad (19)$$

which is equivalent to (18) by the polytopic representation assumption.

Remark 3. Since the dominant eigenvalue of A_ρ is less than Ω_{max}, and through the relation between the eigenvalue and time performance, one can translate this theorem to a time constraint (that is the third objective of the FD observer design).

4. $\mathcal{H}_-$ WITH LOOP SHAPING FILTER DESIGN

It is easy to show that for strictly proper systems, where $F_f = 0$ or not of full row rank, the $\mathcal{H}_-$ index is always zero. In fact:

$$\lim_{\omega\to\infty} T_{rf_\rho}(j\omega) = \lim_{\omega\to\infty}(C(j\omega I - A_\rho^*)^{-1}E_{f_\rho}^* + F_f) = F_f \quad (20)$$

This is also expressed in the LMI (16), the condition of the problem feasibility is that all diagonal terms are negative. In particular:

$$\Psi_{f_\rho} = -F_f^TF_f + \beta^2 I < 0$$

The maximum value of β is:

$$\beta = \sqrt{F_f^TF_f} \quad (21)$$

Thus in strictly proper system, the $\mathcal{H}_-$ strategy cannot be used. This is the case of actuators faults for example. Moreover, even for just proper systems where $F_f \neq 0$, the smallest gain over all frequency range β will always be restricted to the relation (21) regardless the choice of L_i.

To avoid this restriction, one solution have been proposed in (Liu et al., 2005; Wang et al., 2007) and extended in (Farhat and Koenig, 2014).

In order to enhance the residuals to fault sensitivity in low frequencies, i.e. under Assumption 2, one can add an auxiliary direct channel to the system, and then multiply by a high pass filter $\mathcal{F}_H$ as it is shown in figure (1).

For $F_f = \begin{bmatrix} F_{f1} \\ 0 \end{bmatrix}$, a suitable matrix is $F_{f_{add}} = \begin{bmatrix} 0 \\ \sigma I \end{bmatrix}$.

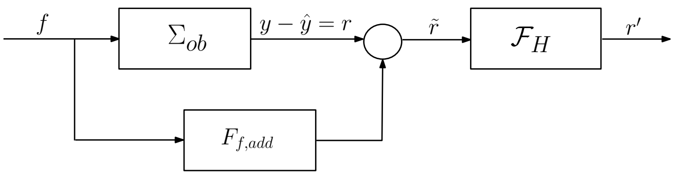

Fig. 1. $\mathcal{H}_-$ Loop Shaping with Additive filter

The high pass filter $\mathcal{F}_H$ is a weighting filter that is used to raise up the high-frequency response, so the minimum singular value of the whole system occurs near the low-frequency region. This high pass filter is designed as:

$$F_H(s) := \left[\begin{array}{c|c} A_h & B_h \\ \hline C_h & D_h \end{array}\right] \quad (22)$$

The parameters A_h, B_h, C_h, and D_h are chosen such that the transfer function $T_{rf}(s)$ has the desired shape. This procedure is analog to the loop shaping method in the standard $\mathcal{H}_\infty$ problem.

By interconnecting the filter with the observer, the systems becomes:

$$\begin{cases} \dot{\hat{x}} &= A_\rho \hat{x} + B_\rho u + L_\rho(y - \hat{y}) \\ \hat{y} &= C\hat{x} + D_\rho u \\ \tilde{r}_\rho &= (y - \hat{y}) + F_{f_{add}} f \\ \dot{x}_h &= A_h x_h + B_h \tilde{r} \\ r &= C_h x_h + D_h \tilde{r} \end{cases} \tag{23}$$

From equations (23) and using the formulation in (6), an augmented residual generator is deduced:

$$\begin{cases} \begin{bmatrix} \dot{\tilde{x}} \\ \dot{x}_h \end{bmatrix} &= A^*_{a_\rho}\tilde{x} + E^*_{ad_\rho} d + E^*_{af_\rho} f \\ r &= C_a \tilde{x} + D_{ad} d + D_{af} f \end{cases} \tag{24}$$

where

$$A^*_{a_\rho} = \begin{bmatrix} A_\rho - L_\rho C & 0 \\ B_h C & A_h \end{bmatrix}, C_a = [D_h C \;\; C_h]$$

$$E^*_{af_\rho} = \begin{bmatrix} E_{f_\rho} - L_\rho F_f \\ B_h(F_f + F_{f_{add}}) \end{bmatrix}, E^*_{ad_\rho} = \begin{bmatrix} E_{d_\rho} - L_\rho F_d \\ B_h F_d \end{bmatrix}$$

$$D_{af} = D_h(F_f + F_{f_{add}}), \quad D_{ad} = D_h F_d$$

Theorems (1) - (3) are then applied to the augmented system (24). That leads to the following theorem:

Remark 4. Since the system (24) is affine in A_h, B_h, C_h, and D_h, the filter F_H could be designed to be parameter dependent, i.e.: different $A_{h,i}, B_{h,i}, C_{h,i}$, and $D_{h,i}$ matrices $\forall i \in \mathbb{S}$ for each vertex i.

Theorem 4. Consider $\mathcal{H}_-/\mathcal{H}_\infty$/pole assignment fault detection observer for the augmented system in (24), for given positive real scalars γ, β and $\Omega_{\max}$, and $\forall i \in \mathbb{S}$, there exist matrices U_{a_i} and a symmetric matrix $P_a > 0$ such that the following inequalities hold:

$$\left[\begin{array}{c|c} P_a A_{0_i} + U_{a_i} C_0 & \\ +A_{0_i}^T P_a + C_0^T U_{a_i}^T & P_a B_{d0_i} + U_{a_i} F_d \\ +C_a^T C_a & +C_a^T D_{ad} \\ \hline \star & D_{ad}^T D_{ad} - \gamma^2 I \end{array}\right] < 0 \tag{25}$$

$$\left[\begin{array}{c|c} -P_a A_{0_i} - U_{a_i} C_0 & -P_a B_{f0_i} \\ -A_{0_i}^T P_a - C_0^T U_{a_i}^T & -U_{a_i} F_f \\ +C_a^T C_a & +C_a^T D_{af} \\ \hline \star & D_{af}^T D_{af} - \beta^2 I \end{array}\right] > 0 \tag{26}$$

$$P_a A_{0_i} + U_{a_i} C_0 + A_{0_i}^T P_a + C_0^T U_{a_i}^T + 2\Omega_{max} P_a < 0 \tag{27}$$

Then the observer gains at each $i - th$ vertex are $L_i = I_0^T (P_a)^{-1} U_{a_i}$.

where the matrices $A_{0_i}, B_{f0_i}, B_{d0_i}, C_0$, and I_0 are:

$$A_{0_i} = \begin{bmatrix} A_i & 0 \\ B_h C & A_h \end{bmatrix}, \quad C_0 = [C \;\; 0], I_0 = \begin{bmatrix} -I \\ 0 \end{bmatrix},$$

$$B_{f0_i} = \begin{bmatrix} E_{f,i} \\ B_h(F_f + F_{f_{add}}) \end{bmatrix}, \quad B_{d0_i} = \begin{bmatrix} E_{d,i} \\ B_h F_d \end{bmatrix}.$$

Proof 4. Only the calculation to get inequality (25) are given here, the same steps are used to find (26).

Apply *Theorem 1* to the augmented system G_{ad_ρ}. The deduced inequality is:

$$\left[\begin{array}{c|c} P_a A_{a_i} + C_a^T C_a & P_a B_{ad_i} \\ +A_{a_i}^T P_a & +C_a^T D_{ad} \\ \hline \star & D_{ad}^T D_{ad} - \gamma^2 I \end{array}\right] < 0 \tag{28}$$

Then decomposing A_{a_i} and B_{ad_i} it follows:

$$P_a A_{a_i} = P_a \begin{bmatrix} A_i & 0 \\ B_h C & A_h \end{bmatrix} + P_a \begin{bmatrix} -I \\ 0 \end{bmatrix} L_i [C \;\; 0]$$

$$P_a B_{ad_i} = P_a \begin{bmatrix} E_{d,i} \\ B_h F_d \end{bmatrix} + P_a \begin{bmatrix} -I \\ 0 \end{bmatrix} L_i F_d$$

Therefore, (28) becomes:

$$\left[\begin{array}{c|c} P_a(A_{0_i} + I_0 L_i C_0) & P_a(B_{d0_i} \\ +(A_{0_i} + I_0 L_i C_0)^T P_a & +I_0 L_i F_d) \\ +C_a^T C_a & +C_a^T D_{ad} \\ \hline \star & D_{ad}^T D_{ad} - \gamma^2 I \end{array}\right] < 0 \tag{29}$$

And by replacing $P_a I_0 L_i$ by U_{a_i}, the BMI becomes the following LMI:

$$\left[\begin{array}{c|c} P_a A_{0_i} + U_{a_i} C_0 & \\ +A_{0_i}^T P_a + C_0^T U_{a_i}^T & P_a B_{d0_i} + U_{a_i} F_d \\ +C_a^T C_a & +C_a^T D_{ad} \\ \hline \star & D_{ad}^T D_{ad} - \gamma^2 I \end{array}\right] < 0 \tag{30}$$

□

To resume, the algorithm to design the 3-objectives FD observer , at each vertex, is:

(1) Choose the parameter σ in additive term $F_{f_{add}}$, it should have small value ($\sigma = 0.1$),
(2) Choose the filter's F_H parameters: A_h, B_h, C_h, and D_h such that the filters bandwidth is higher than the system's bandwidth.
(3) Choose Ω_{max} such that the observers dynamics (rise time) meets the desired specifications.
(4) Choose a value for γ, such that $\gamma < 1$.
(5) Compute β and L_i solution to the optimization problem formulated by LMIs (25), (26) and (27).
(6) repeat steps 4) and 5) by minimizing γ till a compromise between γ and β is reached.

Remark 5. It is to note that in the augmented system, the matrix P_a is of dimension $n+n_H$, with n_H is the dimension of the weighting filter $\mathcal{F}_H$.

Remark 6. As described in the algorithm of the filter design, the LMIs (25), (26) and (27) are solved using iterative algorithm.

In fact, only LMIs (26) are subject to the optimization criterion $max(\beta)$, while LMIs (25) and (27) are only tested for their feasibility i.e. for given scalars Ω_{max} and γ that are manually set over iterations: $\gamma(k+1) < \gamma(k)$, k being the k^{th} step of iteration.

Using this iterative bisection method transforms the joint H_-/H_∞ optimization problem that might be non-convex into tow separate problems, whereas convexity is guaranteed.

It is to be pointed that the resulting values of the couple (γ, β) is a local optimum of the $\mathcal{H}_-/\mathcal{H}_\infty$ problem, since it depends on the initial values of γ, and the steps of minimization.

5. EXAMPLE

Consider the problem of the FD in the lateral control of a vehicle. Some experimental data have been taken from a real "Renaul Scenic", provided by the french laboratory MIPS (Mulhouse).

The widely used bicycle-model is a good representation of the system (Mammar and Koenig, 2002). However, this model is non-linear since it has $\frac{1}{v}$ and $\frac{1}{v^2}$ terms in it:

$$\begin{bmatrix} \dot{\beta}(t) \\ \ddot{\psi}(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{c_r + c_f}{mv(t)} & \dfrac{c_r l_r - c_f l_f}{mv^2(t)} - 1 \\ \dfrac{c_r l_r - c_f l_f}{I_z} & -\dfrac{c_r l_r^2 + c_f l_f^2}{I_z v(t)} \end{bmatrix} \begin{bmatrix} \beta(t) \\ \dot{\psi}(t) \end{bmatrix} + \begin{bmatrix} \dfrac{c_f}{mv(t)} \\ \dfrac{c_r l_f}{I_z} \end{bmatrix} u_L(t) + \begin{bmatrix} \dfrac{1}{mv} \\ \dfrac{l_w}{I_z} \end{bmatrix} F_w(t)$$

$$y = \begin{bmatrix} -\dfrac{c_r + c_f}{m} & \dfrac{c_f l_f - c_r l_r}{mv^2(t)} \end{bmatrix} \begin{bmatrix} \beta(t) \\ \dot{\psi}(t) \end{bmatrix} + \begin{bmatrix} \dfrac{c_f}{m} \end{bmatrix} u_L(t)$$

The measured output is the lateral acceleration γ_L, the entry command is the steering angle u_L, the states are the side slip angle β and the yaw rate $\dot{\psi}$, and we consider the wind force as an unknown perturbation signal F_w.

The fault considered in this application is an actuator fault, that occurs on the actuator. Let $\rho_1 = \frac{1}{v}$ and $\rho_2 = \frac{1}{v^2}$, the LPV state space representation in this case becomes :

$$\begin{cases} \dot{x} = A(\rho)x + B(\rho)(u + f) + E_d(\rho)d \\ y = C(\rho)x + D(\rho)(u + f) \end{cases} \tag{31}$$

Here, only two parameters are variant and they affect matrices A, B, E_d and C, then:

$$\begin{aligned} A(\rho) &= A_0 + \rho_1 A_1 + \rho_2 A_2 \\ B(\rho) &= B_0 + \rho_1 B_1 \\ E_d(\rho) &= E_{d,0} + \rho_1 E_{d,1} \\ C(\rho) &= C_0 + \rho_2 C_1 \end{aligned} \tag{32}$$

Since the output matrices are parameter dependent, the transformation described in *Remark 1* can be performed.

In this example, a velocity range is considered: $v(t)$ varies between 20 and 40 km/h. A polytope of $2^N = 4$ vertices can be constructed:

$$\begin{aligned} \omega_1 &= \begin{bmatrix} \underline{\rho}_1 & \underline{\rho}_2 \end{bmatrix}, & \omega_2 &= \begin{bmatrix} \overline{\rho}_1 & \underline{\rho}_2 \end{bmatrix}, \\ \omega_3 &= \begin{bmatrix} \overline{\rho}_1 & \overline{\rho}_2 \end{bmatrix} & , \omega_4 &= \begin{bmatrix} \underline{\rho}_1 & \overline{\rho}_2 \end{bmatrix}. \end{aligned} \tag{33}$$

Now, noticing the relation between the varying parameters: $\rho_2 = \rho_1^2$, the vertex $\omega_4 = \begin{bmatrix} \underline{\rho}_1 & \overline{\rho}_2 \end{bmatrix}$ is not required since the 3 other vertices are sufficient to characterize the parameter definition. Then the polytope could be reduced to only three vertices: ω_1, ω_2 and ω_3. This method has been considered in Robert et al. (2010)

The polytopic coordinates could be obtained by solving the following matrix equality:

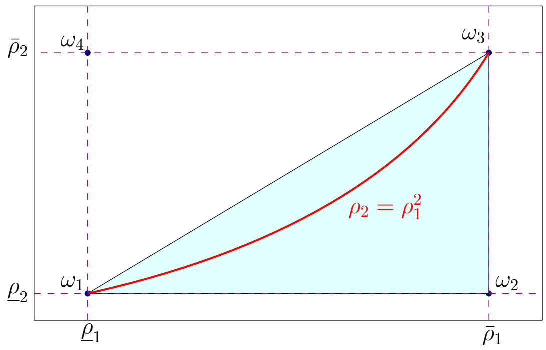

Fig. 2. Illustration of the polytope reduction

$$\begin{bmatrix} \underline{\rho}_1 & \overline{\rho}_1 & \overline{\rho}_1 \\ \underline{\rho}_2 & \underline{\rho}_2 & \overline{\rho}_2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha^1 \\ \alpha^2 \\ \alpha^3 \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ 1 \end{bmatrix} \tag{34}$$

In this case, the resulting residual is calculated according to (4):

$$r = \alpha^1 r_1 + \alpha^2 r_2 + \alpha^3 r_3 \tag{35}$$

where r_i is the residual obtained from the observer at vertex ω_i

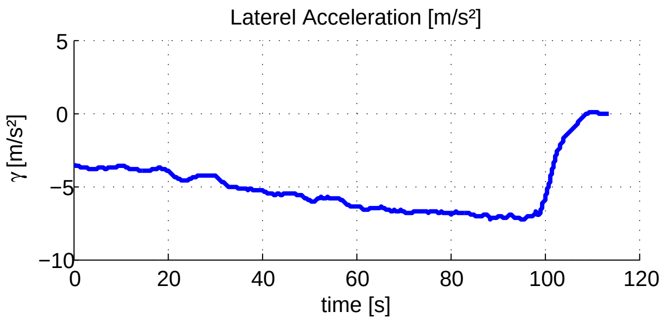

Fig. 3. System output: Lateral acceleration $[\mathrm{m/s}^2]$

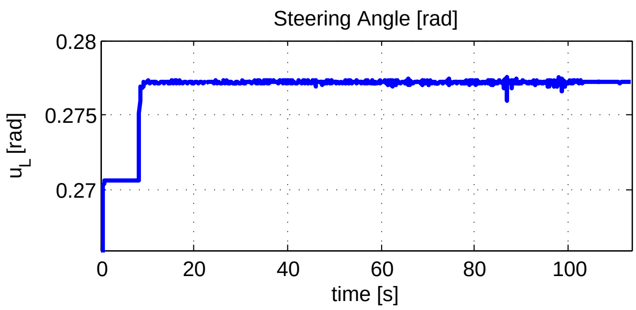

Fig. 4. System command input: Steering Angle [rad]

Applying the algorithm for the design of the observer:

First, define the loop shaping filter: suitable $F_{f_{add}}$ matrix is chosen for the additive filter. Let

$$F_{f_{add}} = [.06],$$

and

$$F_{H,1}(s) = F_{H,2}(s) = \left(\frac{s/.05 + 1}{s/7 + 1} \right)^2$$

Then, choose a value of poles assignment for a time response of less than 0.5 second: $\Omega_{max} = -6$

And following the steps 3) to 5), the resulting LMIs as detailed in section IV are solved using SeDuMi and the LPV fault detection observer can be designed, meeting the $\mathcal{H}_-/\mathcal{H}_\infty$ and time constraints objectives.

The result of the design is given in figure 6. For a fault that occurs between $t = 43$ and $t = 48$ s, the residuals raises alarming a fault detection.

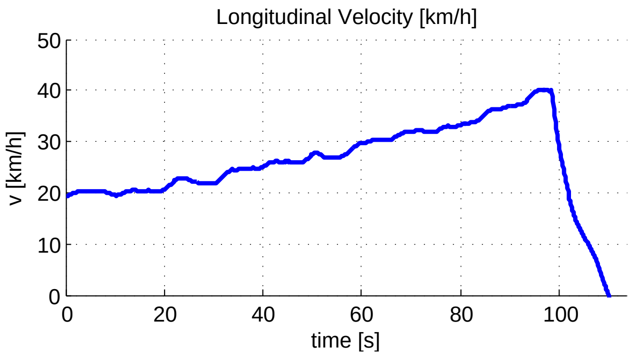

Fig. 5. Longitudinal velocity [km/h]

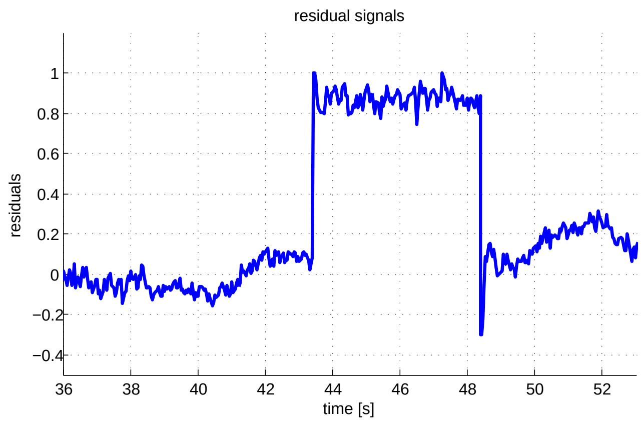

Fig. 6. Residuals signal

6. CONCLUSION AND FURTHER WORK

The technique presented in this paper provides a framework for generating a class of fault detection observers for LPV systems.

Several time- and frequency-domain specifications have been expressed as LMI constraints on the observers' state-space matrices. These analyses are then used for multi-objective synthesis purposes. A compromise of these objectives is proposed as a criterion to minimize. It is formulated an LMIs feasibility problem. The solution of the optimization problem can be found by using efficient LMI solver. An example with real data is given to validate this approach.

In future work, this design can be be applied to critical situation detection in lateral vehicle dynamics. The ideas presented here can be generalized for uncertain LPV systems, and quasi LPV system. LPV-Fault tolerant control is also one possible extension of this work.

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